Concrete Repair, Rehabilitation and Retrofitting – Alexander (eds.)
© 2006 Taylor & Francis Group, London, ISBN 0 415 39654 9

Software for probability-based durability analysis of concrete structures

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ABSTRACT: In recent years, much research work has been carried out in order to obtain a more controlled durability and long-term performance of concrete structures in chloride containing environments. In particular, the development of new procedures for probability-based durability design has been shown to provide a more realistic basis for the analysis. Although relevant data is still lacking, this approach has been successfully applied to several new concrete structures, where requirements for a more controlled durability and service life have been specified. A probability-based durability analysis has also become an important and integral part of condition assessment of existing concrete structures in chloride containing environments. In order to facilitate the probability-based durability analysis, simple software has been developed, where the probabilistic approach is based on a Monte Carlo simulation.

In the present paper, the software for the probability-based durability analysis is briefly described and applied in order to demonstrate the importance and sensitivity of the various durability parameters affecting and controlling the durability of concrete structures in chloride containing environments.

1 INTRODUCTION

Since all parameters both for concrete durability and environmental exposure typically show a high scatter, a probability-based approach provides a very powerful basis for durability analysis [1–3]. This approach is primarily applied in order to obtain more controlled durability and long-term performance of new concrete structures, but it also provides a very valuable basis for condition assessment of existing concrete structures in chloride containing environments. For the probability approach, a number of sophisticated statistical methods exist which may be adopted for the durability analysis. However, as relevant input data is still lacking for an analysis and as a number of assumptions have to be made, a very simple software program based on a Monte Carlo simulation has been developed.

In what follows, the software program is briefly described and used in order to demonstrate the importance and sensitivity of the various durability parameters affecting and controlling the durability of concrete structures in a chloride containing environment.

2 MODEL DESCRIPTION

For the software program DURACON [4, 5], the modelling of chloride penetration and time to depassivation is based on Fick's Second Law of Diffusion, in combination with a time dependent diffusion coefficient [6]. The software incorporates the stochastic nature of the individual durability parameters which are needed as input to the program. These durability parameters include the diffusion coefficient, which may either be obtained from accelerated laboratory testing or curve fitting of chloride profiles from existing concrete structures; the time dependence of the diffusion coefficient; and the critical chloride content for depassivation of embedded steel, both of which may be obtained from existing literature or other experience for the given type of cement and concrete. The concrete cover and the environmental exposure expressed in the form of surface chloride concentration, (obtainable from either measurements or previous experience) are also important durability and input parameters for the software program.

As time to depassivation and onset of steel corrosion is used as a basis for the serviceability limit state, the program can express the probability of failure or risk for the serviceability limit state to be reached after a certain period of time. For new concrete structures, this provides an appropriate basis for establishing overall durability criteria for the structure in question [7]. For existing concrete structures, where the chloride front has still not reached the embedded steel, the program can be used for estimating the probability of corrosion after a certain period of time [8].

2.1 *Rate of chloride penetration*

According to Fick's Second Law of Diffusion, we have the following expression:

$$\frac{dC(x,t)}{dt} = D_C \cdot \frac{d^2C(x,t)}{dx^2} \tag{1}$$

where $C(x,t)$ is the chloride ion concentration at a distance x from the concrete surface after being exposed for a period of time t, and D_C is the chloride diffusion coefficient. By solving this equation for predefined boundary conditions, the following equation is obtained:

$$C(x,t) = C_S\left[1 - erf\left(\frac{x}{2\sqrt{D.t}}\right)\right] \tag{2}$$

where C_S is the chloride ion concentration on the concrete surface, and *erf* is the error function.

The time dependence of the diffusion coefficient is normally expressed as:

$$D(t) = D_0 \cdot \left(\frac{t}{t_0}\right)^{\alpha} \tag{3}$$

where D_0 is the diffusion coefficient at a given time t_0, and the exponent α represents the time dependence of the diffusion coefficient or the increased ability of the concrete to resist chloride penetration over time.

By substituting Eq. 3 into Eq. 2, an expression is obtained that permits the prediction of chloride penetration based on the time dependent diffusion coefficient, given by:

$$c_X = c_S\left[1 - erf\left(x/2.\sqrt{D_0 \cdot t \cdot (t/t_0)^{\alpha}}\right)\right] \tag{4}$$

2.2 *Probabilistic approach*

The probabilistic approach is based on the Monte Carlo Method, which can be briefly described as a statistical simulation method, where sequences of random numbers are applied to perform the simulation. In the present application of the simulation, the physical process is simulated directly by use of the modified Fick's Second Law of Diffusion for describing the transport process. The only requirement is that all the input parameters to the equation be described by a probability density function. Once the probability density functions of the various durability parameters of the system are known, the probability of failure is based on the evaluation of the limit state function for a large number of trials.

When a simulation method is used for calculating the probability of failure, the failure function is calculated for each outcome. If the outcome is in the failure region, then the contribution to the probability of failure is obtained. The probability of failure is estimated by the following expression:

$$p_f = \frac{1}{N} \cdot \sum_{j=1}^{N} I\left[g\left(r_j, s_j\right)\right] \tag{5}$$

where N is the number of simulations, $I\,[g(r_j,s_j)]$ is the indicator function and $g(r_j,s_j)$ is the limit state equation, where s represents the environmental load and r is the resistance of the concrete against chloride penetration.

The standard error of the probability of failure is estimated by [9]:

$$s = \sqrt{\frac{p_f\left(1 - p_f\right)}{N}} \tag{6}$$

Since the accuracy of the Monte Carlo Method depends mainly on the number of trials undertaken [10] and the method is easy to implement, a simulation based on this method appears to be both simple and intuitive.

2.3 *Assumptions and limitations*

It should be noted that the rate of chloride penetration may also be controlled by other mechanisms such as capillary suction or crack penetration. However, based on current knowledge, it is assumed that diffusion is a dominating transport process for chloride penetration into concrete. Since the rate of diffusion is also controlled by temperature, however, and the current software does not take temperature into consideration, this may also represent a limitation. This may not be so important for a durability analysis of an existing structure, since the input parameter for the diffusion coefficient D_0 then will also reflect the prevailing temperature conditions for the given structure. This may also be the case for the input parameter α, which reflects the time dependence of the diffusion coefficient. However, there is still a lack of relevant data and information on the various input parameters. Therefore, a critical interpretation of obtained results and sound engineering judgement are important for a proper utilization of the software.

3 SENSITIVITY ANALYSIS

3.1 *Method of sensitivity analysis*

The objective of the sensitivity analysis is to observe the variation of the probability of corrosion for each

Table 1. Range of parameter values for sensitivity analysis.

Parameters	Base case	Range of values
D_0 (10^{-12} m^2/s)	N(2.00, 0.20)	1.0, 3.0, 4.0, 5.0
c_{CR} (% wt. of conc.)	N(0.30, 0.03)	0.6, 0.8, 1.2
x_C (mm)	N(50.0, 5.0)	40, 55, 60, 70
c_S (% wt. conc.)	N(3.4, 034)	2.4, 3.0, 3.8, 4.3
t/t_0 (years/days)	50/28	Not varied
α	N(0.35, 0.035)	Not varied

individual durability parameters, after a service period of 50 years. The analysis was performed by varying the parameters over a relevant range of values while maintaining the other parameters constant. The parameters used were: diffusion coefficient at time t_0, D_0; critical chloride ion concentration, c_{CR}; concrete cover of reinforcement, x_C; surface chloride ion concentration, c_S; exposure period to the chloride containing environment, t; the age at which the diffusion coefficient D_0 was determined, t_0; and the exponent for time dependence of diffusion coefficient, α. For comparative purposes, a base case was also defined; its values together with the range of the parameters analysed are shown in Table 1.

3.2 *Sensitivity of durability parameters*

If the diffusion coefficient increases, as seen in Figure 1, so does the probability of corrosion. This should be expected, since the larger the diffusion coefficient, the more penetrable the concrete is. For the base case values chosen, however, it is clear that the probability of corrosion can be reduced by more than 95% if the diffusion coefficient is reduced from 5.0×10^{-12} m^2/s to 1.0×10^{-12} m^2/s, and hence, the diffusion coefficient appears to be a very sensitive parameter. A more efficient way of controlling the diffusion coefficient is the proper selection of cement type as mentioned in 4.3 and shown in Figure 5.

As the concentration of chloride ions needed for corrosion increases, Figure 2 shows that the probability of failure rapidly decreases up to a level of 0.10% by weight concrete. For most types of binder system, therefore, the analysis indicates that the critical level of chloride concentration is a highly sensitive durability parameter.

Figure 3 demonstrates that when the surface chloride concentration increases, there is linear increase in probability of corrosion. This is further to be expected as a high chloride concentration on the concrete surface will yield a high driving gradient for the chloride diffusion.

In Figure 4, the sensitivity of varying concrete covers for the probability of corrosion is shown. For

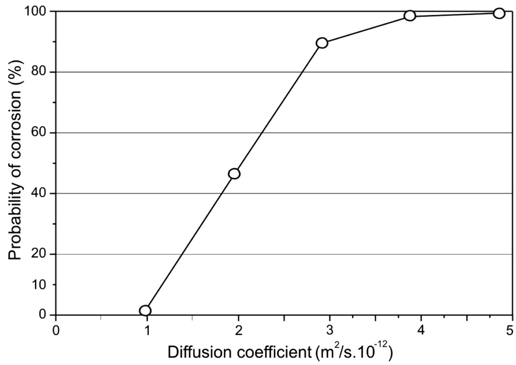

Figure 1. Effect of the diffusion coefficient on probability of corrosion.

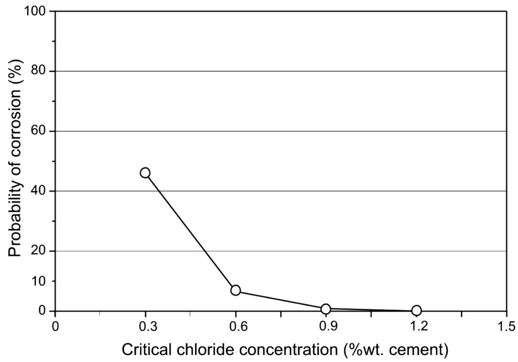

Figure 2. Effect of critical chloride concentration on probability of corrosion.

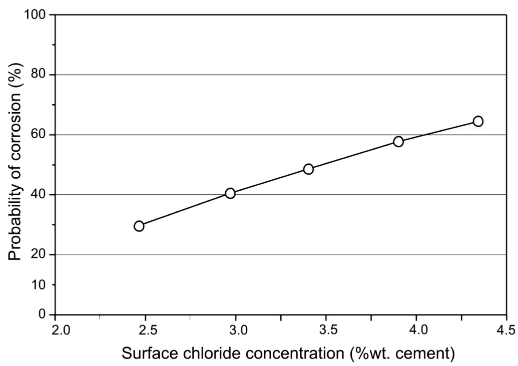

Figure 3. Effect of surface chloride concentration on probability of corrosion.

decreasing concrete cover below 60 mm, it appears that the probability of corrosion rapidly increases in a linear manner consequently showing that concrete cover is also a very sensitive durability parameter.

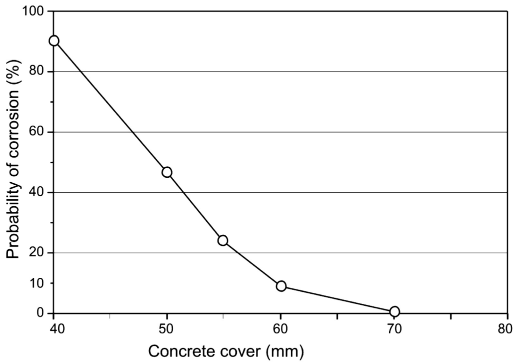

Figure 4. Effect of concrete cover on probability of corrosion.

4 DURABILITY DESIGN OPTIONS

4.1 *General*

In order to demonstrate how the DURACON program [4] can be used to select appropriate concrete mixtures or concrete cover in order to obtain a more controlled durability; the effects of three different concrete variables on the durability analysis are set out below. In the first example, the cement type used in the concrete mixture was investigated. In the second example, the effect of varying cement content was studied, while in the third example, the concrete cover was varied.

4.2 *Input durability parameters*

As input to the durability analysis, the following durability parameters were used:

- t – exposure period to the chloride containing environment. In all cases, an exposure period of 50 years was selected.
- c_S – surface chloride concentration. In all cases, a normal distribution of surface chloride concentration with an average of 5.4% by weight of cement and a coefficient of variation (CoV) of 10% were adopted.
- x_C – concrete cover of reinforcement. In the first two cases, a normal distribution of concrete cover with an average of 50 mm and a 5 mm standard deviation were adopted.
- D_0 – diffusion coefficient at time t_0. For each concrete mixture, this coefficient was determined based on accelerated laboratory testing [11], assuming a normal distribution.
- t_0 – the age at which the diffusion coefficient D_0 was determined. For all types of concrete, this age was 28 days.
- c_{CR} – critical chloride ion concentration. Based on existing experience, a normally distributed value averaging 0.48 and standard deviation of 0.15 were assumed.
- α – exponent for time dependence of diffusion coefficient. Based on existing experience for each type of cement, appropriate values for both the exponent and CoV were assumed.

Table 2. Parameter values for different types of cement.

Cement type	D_0 (10^{-12} m^2/s)	α
CEMI	*N(10.5; 0.66)	N(0.37;0.07)
CEM III/B	N(5.3; 0.59)	N(0.60;0.15)
CEM II/A-V	N(10.1; 0.81)	N(0.51;0.07)
CEM I + CSF	N(4.7; 0.51)	N(0.39;0.07)

* N- normal distribution (average, standard deviation)

4.3 *Effect of cement type*

In order to study the effect of cement type, four concrete mixtures with four different types of cement were produced. Based on four types of cement some concrete test mixtures were produced in order to test the chloride diffusivity [11]. Three types of cement included a high-performance portland cement (CEM I 52.5), a blended fly-ash cement (CEM II/A-V 42.5) and a blast-furnace slag cement with approximately 70% slag (CEM III/B 42.5). The test mixtures had a cement content of 420 kg/m^3 and a w/c ratio of 0.45. The forth cement was a high-performance portland cement (CEM I 52.5) mixed with 10% CSF. The test mixture had a cement content and silica fume content of 390 kg/m^3 and 39 kg/m^3, respectively. The w/c ratios for the mixtures were 0.38 and 0.35 for k-factors of 1 and 2, respectively.

The effect of cement type on the observed diffusion coefficient (m^2/s) is shown in Table 2, where the adopted values for α are also included.

As can be seen from Figure 5, the type of cement had a significant effect on the probability of corrosion. Figure 1 clearly demonstrates the big difference in resistance against chloride penetration between the blast furnace slag cement and the pure portland cement, which is in accordance with previous experience [12]. For the pure portland cement, the probability level of 10% for corrosion would be exceeded within a period of approximately 3 years, while for the blast furnace slag cement, this level of risk of corrosion would only be exceeded within a period of 32 years. For the fly ash cement and the combination of the portland cement with silica fume, the corresponding risk for corrosion would be exceeded within a period of approximately 9 and 11 years, respectively.

4.4 *Effect of cement content*

In order to study the effect of cement content, four different types of concrete with constant water to

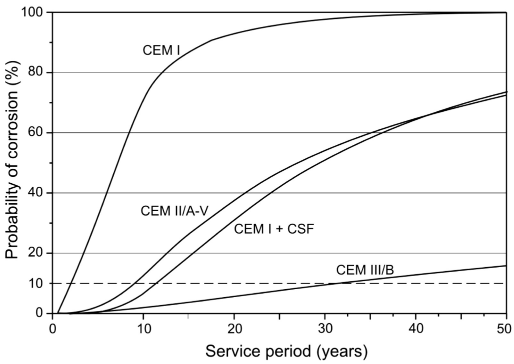

Figure 5. Effect of cement type on the probability of corrosion.

Table 3. Parameter values for varying cement content.

Cement content (kg/m^3)	D_0 (10^{-12}m^2/s)
300	*N(15.7, 1.57)
350	N(11.70, 1.17)
370	N(8.00, 0.80)
400	N(5.50, 0.55)

* N- normal distribution (average, standard deviation)

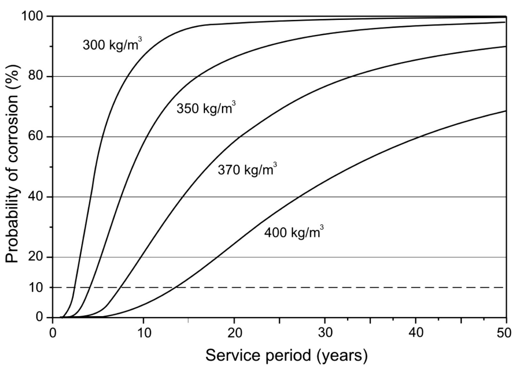

Figure 6. The effect of cement content on the probability of corrosion.

cement ratio but a varying content of CEM I were produced. The effect of the cement content on the observed diffusion coefficients is shown in Table 3. As can be seen from Figure 6, the increasing cement content distinctly reduced the probability of corrosion at any given time of exposure.

It is also important to note that the smaller the cement content, the more rapidly the probability of

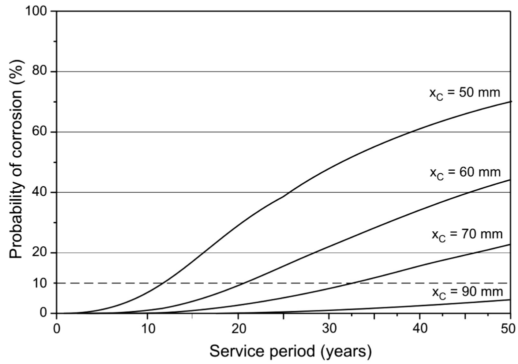

Figure 7. Effect of concrete cover on the probability of corrosion.

corrosion increased. In most codes for reliability of structures, however, an upper level of 10% for probability of failure is given as the accepted standard [7]. Regardless of cement content, Figure 6 demonstrates that a probability level of 10% for corrosion will already be exceeded within a period of approximately 3 to 14 years. Therefore, another type of cement should be used for concrete structures in chloride containing environment.

4.5 *Effect of concrete cover*

Based on the data previously shown for the portland cement with 10% silica fume concrete mix, (see example in Table 2), a new durability analysis was carried out in order to find out the effect of increased concrete cover above the minimum requirement of 50 mm (Figure 7). The concrete depths of 50, 60, 70 and 90 mm were used.

As can be seen from Figure 7, the concrete cover is also of great importance for the probability of corrosion. While a nominal concrete cover of 50 mm only would give a service period of approximately 10 years, an effective, nominal concrete cover of 90 mm would give a service period of more than 50 years.

Since it may be difficult to significantly increase the concrete cover in the lower part of deck beams beyond 70 mm, only a partial replacement of the conventional reinforcement with stainless steel reinforcement could be used in order to obtain a substantial increase of the effective concrete cover beyond 70 mm.

5 CONCLUSIONS

For a probabilistic approach to durability analysis, several sophisticated statistical methods exist which may be adopted for the analysis. However, since there is still a lack of relevant input data for an analysis and a number of assumptions have to be made, a very simple software program based on a Monte Carlo simulation has been developed, some results of which are demonstrated in the present paper.

As part of the durability design options, it was shown that increased contents of a pure portland cement did not drastically change the probability of chloride-induced corrosion compared to that of selecting a more adequate type of cement. Thus, for the pure portland cement, a probability level of 10% for corrosion would be exceeded within a period of approximately 4 years, while for the blast furnace slag cement, such a risk of corrosion would only be exceeded within a period of 32 years. For the fly ash cement and the combination of the portland cement with silica fume, the corresponding risk of corrosion would be exceeded within a period of approximately 9 and 11 years, respectively. In the sensitivity analysis of the various durability parameters, it was further shown that a reduced chloride diffusivity of the concrete from $5.0 \times 10^{-12} m^2/s$ to $1.0 \times 10^{-12} m^2/s$ would reduce the probability of corrosion by more than 95% over a service period of 50 years. Hence, the chloride diffusivity of a concrete is a very sensitive durability parameter for concrete structures in chloride containing environments.

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